PILOT LEARNING
CALCULUS II ENGINEERING
PROBLEM-SET 8
FALL 2019

(1) Find a formula for the general term an of the sequence, assuming that the pattern of the first few terms continues.
   (a) $\{5, 8, 11, 14, 17, \dots\}$
   (b) $\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \dots\}$
   (c) $\{1, 0, -1, 0, 1, 0, -1, 0, \dots\}$

(2) Determine whether or not the sequences a_n converge, and if so find the limit:
   (a) $a_n = \frac{8n-7}{7n+8}$
   (b) $a_n = 10 - (0.99)^n$
   (c) $a_n = \frac{(\ln n)^3}{n^2}$
   (d) $a_n = \frac{n-e^n}{n+e^n}$
   (e) $a_n = \cos\frac{n}{2}$
   (f) $a_n = \frac{(2n-1)!}{(2n+1)!}$
   (g) $a_n = 2^{-n}\cos \pi n$
   (h) $a_n = \ln(2n^2+1) - \ln(n^2+1)$
   (i) $a_n = \frac{e^n - e^{-n}}{e^{2n}-1}$

(3) A fish farmer has 5000 catfish in his pond. The number of catfish increases by 10% per month and the farmer harvests 300 catfish per month.
   (a) Show that the catfish population P_n after n months is given recursively by
   $$P_n = 1.1P_{n-1} - 300; \quad P_0 = 5000$$
   (b) By induction or otherwise, show that P_n is decreasing and bounded below by 3000. Apply the Monotonic Sequence Theorem to show that $\lim_{n\to\infty} P_n$ exists.
   (c) Find $\lim_{n\to\infty} P_n$.

(4) A sequence a_n is given by $a_1 = \sqrt{2};\ a_n = \sqrt{2 + a_{n-1}}$
   (a) By induction or otherwise, show that a_n is increasing and bounded above by 2. Apply the Monotonic Sequence Theorem to show that $\lim_{n\to\infty} a_n$ exists.
   (b) Find $\lim_{n\to\infty} a_n$

(5) Find the limit of the sequence $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}, \dots\}$